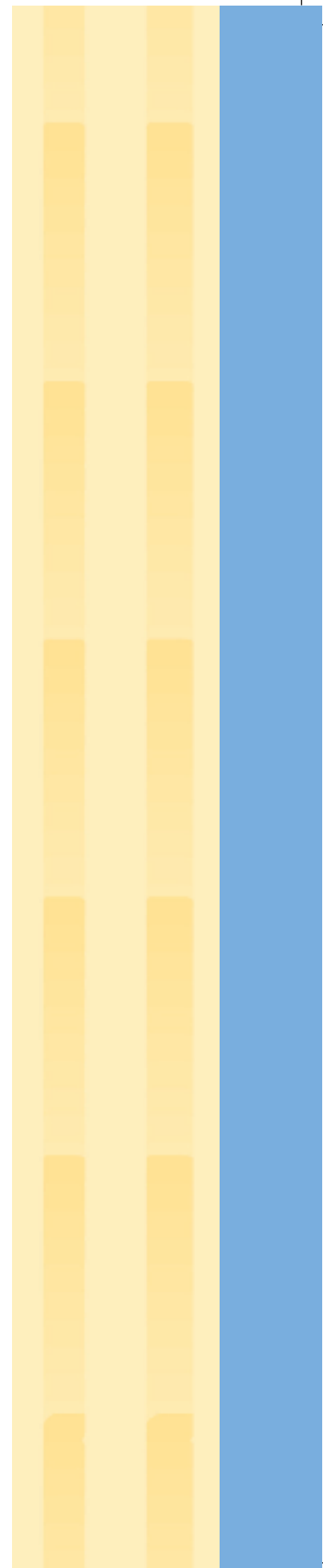


part II

Classical Theory: The Economy in the Long Run



CHAPTER 3 THREE

National Income: Where It Comes From and Where It Goes

A large income is the best recipe for happiness I ever heard of.

—Jane Austen

The most important macroeconomic variable is gross domestic product (GDP). As we have seen, GDP measures both a nation's total output of goods and services and its total income. To appreciate the significance of GDP, one need only take a quick look at international data: compared with their poorer counterparts, nations with a high level of GDP per person have everything from better childhood nutrition to more televisions per household. A large GDP does not ensure that all of a nation's citizens are happy, but it may be the best recipe for happiness that macroeconomists have to offer.

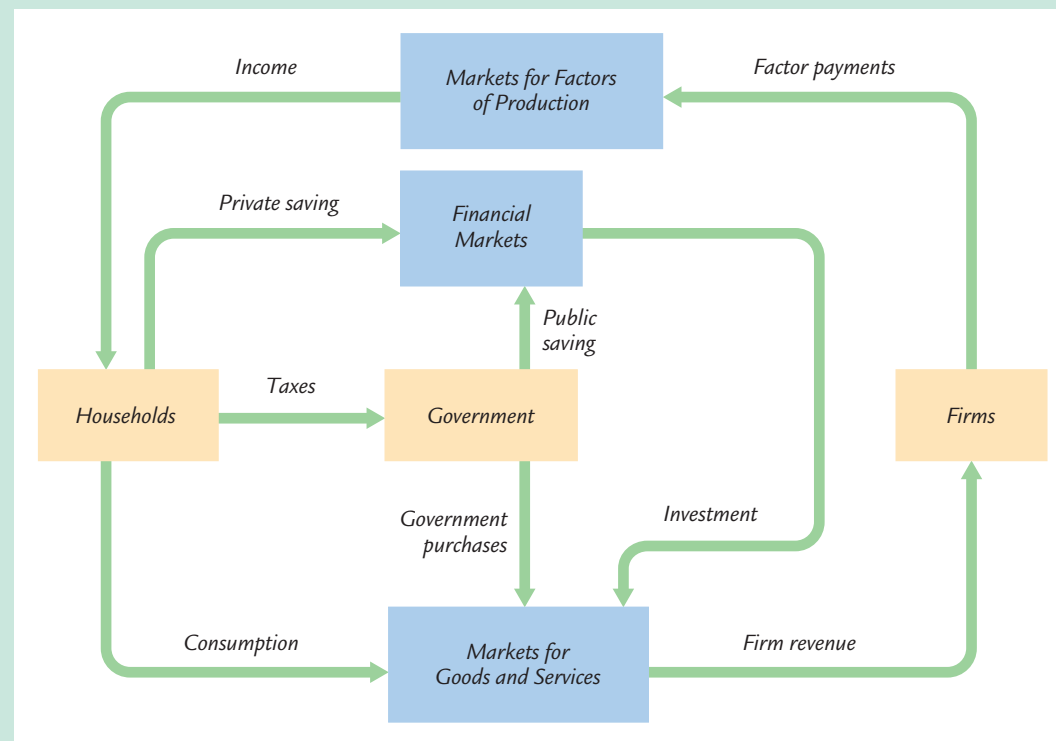
This chapter addresses four groups of questions about the sources and uses of a nation's GDP:

- ▶ How much do the firms in the economy produce? What determines a nation's total income?
- ▶ Who gets the income from production? How much goes to compensate workers, and how much goes to compensate owners of capital?
- ▶ Who buys the output of the economy? How much do households purchase for consumption, how much do households and firms purchase for investment, and how much does the government buy for public purposes?
- ▶ What equilibrates the demand for and supply of goods and services? What ensures that desired spending on consumption, investment, and government purchases equals the level of production?

To answer these questions, we must examine how the various parts of the economy interact.

A good place to start is the circular flow diagram. In Chapter 2 we traced the circular flow of dollars in a hypothetical economy that produced one product, bread, from labor services. Figure 3-1 more accurately reflects how real economies function. It shows the linkages among the economic actors—households, firms,

figure 3-1



The Circular Flow of Dollars Through the Economy This figure is a more realistic version of the circular flow diagram found in Chapter 2. Each yellow box represents an economic actor—households, firms, and the government. Each blue box represents a type of market—the markets for goods and services, the markets for the factors of production, and financial markets. The green arrows show the flow of dollars among the economic actors through the three types of markets.

and the government—and how dollars flow among them through the various markets in the economy.

Let's look at the flow of dollars from the viewpoints of these economic actors. Households receive income and use it to pay taxes to the government, to consume goods and services, and to save through the financial markets. Firms receive revenue from the sale of goods and services and use it to pay for the factors of production. Both households and firms borrow in financial markets to buy investment goods, such as houses and factories. The government receives revenue from taxes and uses it to pay for government purchases. Any excess of tax revenue over government spending is called public saving, which can be either positive (a budget surplus) or negative (a budget deficit).

In this chapter we develop a basic classical model to explain the economic interactions depicted in Figure 3-1. We begin with firms and look at what

determines their level of production (and, thus, the level of national income). Then we examine how the markets for the factors of production distribute this income to households. Next, we consider how much of this income households consume and how much they save. In addition to discussing the demand for goods and services arising from the consumption of households, we discuss the demand arising from investment and government purchases. Finally, we come full circle and examine how the demand for goods and services (the sum of consumption, investment, and government purchases) and the supply of goods and services (the level of production) are brought into balance.

3-1 What Determines the Total Production of Goods and Services?

An economy's output of goods and services—its GDP—depends on (1) its quantity of inputs, called the factors of production, and (2) its ability to turn inputs into output, as represented by the production function. We discuss each of these in turn.

The Factors of Production

Factors of production are the inputs used to produce goods and services. The two most important factors of production are capital and labor. Capital is the set of tools that workers use: the construction worker's crane, the accountant's calculator, and this author's personal computer. Labor is the time people spend working. We use the symbol K to denote the amount of capital and the symbol L to denote the amount of labor.

In this chapter we take the economy's factors of production as given. In other words, we assume that the economy has a fixed amount of capital and a fixed amount of labor. We write

$$K = \bar{K}.$$
$$L = \bar{L}.$$

The overbar means that each variable is fixed at some level. In Chapter 7 we examine what happens when the factors of production change over time, as they do in the real world. For now, to keep our analysis simple, we assume fixed amounts of capital and labor.

We also assume here that the factors of production are fully utilized—that is, that no resources are wasted. Again, in the real world, part of the labor force is unemployed, and some capital lies idle. In Chapter 6 we examine the reasons for unemployment, but for now we assume that capital and labor are fully employed.

The Production Function

The available production technology determines how much output is produced from given amounts of capital and labor. Economists express the available technology using a **production function**. Letting Y denote the amount of output, we write the production function as

$$Y = F(K, L).$$

This equation states that output is a function of the amount of capital and the amount of labor.

The production function reflects the available technology for turning capital and labor into output. If someone invents a better way to produce a good, the result is more output from the same amounts of capital and labor. Thus, technological change alters the production function.

Many production functions have a property called **constant returns to scale**. A production function has constant returns to scale if an increase of an equal percentage in all factors of production causes an increase in output of the same percentage. If the production function has constant returns to scale, then we get 10 percent more output when we increase both capital and labor by 10 percent. Mathematically, a production function has constant returns to scale if

$$zY = F(zK, zL)$$

for any positive number z . This equation says that if we multiply both the amount of capital and the amount of labor by some number z , output is also multiplied by z . In the next section we see that the assumption of constant returns to scale has an important implication for how the income from production is distributed.

As an example of a production function, consider production at a bakery. The kitchen and its equipment are the bakery's capital, the workers hired to make the bread are its labor, and the loaves of bread are its output. The bakery's production function shows that the number of loaves produced depends on the amount of equipment and the number of workers. If the production function has constant returns to scale, then doubling the amount of equipment and the number of workers doubles the amount of bread produced.

The Supply of Goods and Services

We can now see that the factors of production and the production function together determine the quantity of goods and services supplied, which in turn equals the economy's output. To express this mathematically, we write

$$\begin{aligned} Y &= F(\bar{K}, \bar{L}) \\ &= \bar{Y}. \end{aligned}$$

In this chapter, because we assume that the supplies of capital and labor and the technology are fixed, output is also fixed (at a level denoted here as \bar{Y}). When we



discuss economic growth in Chapters 7 and 8, we will examine how increases in capital and labor and improvements in the production technology lead to growth in the economy's output.

3-2 How Is National Income Distributed to the Factors of Production?

As we discussed in Chapter 2, the total output of an economy equals its total income. Because the factors of production and the production function together determine the total output of goods and services, they also determine national income. The circular flow diagram in Figure 3-1 shows that this national income flows from firms to households through the markets for the factors of production.

In this section we continue developing our model of the economy by discussing how these factor markets work. Economists have long studied factor markets to understand the distribution of income. (For example, Karl Marx, the noted nineteenth-century economist, spent much time trying to explain the incomes of capital and labor. The political philosophy of communism was in part based on Marx's now-discredited theory.) Here we examine the modern theory of how national income is divided among the factors of production. This theory, called the *neoclassical theory of distribution*, is accepted by most economists today.

Factor Prices

The distribution of national income is determined by factor prices. **Factor prices** are the amounts paid to the factors of production—the wage workers earn and the rent the owners of capital collect. As Figure 3-2 illustrates, the price each factor of production receives for its services is in turn determined by the supply and demand for that factor. Because we have assumed that the economy's factors of production are fixed, the factor supply curve in Figure 3-2 is vertical. The intersection of the downward-sloping demand curve and the vertical supply curve determines the equilibrium factor price.

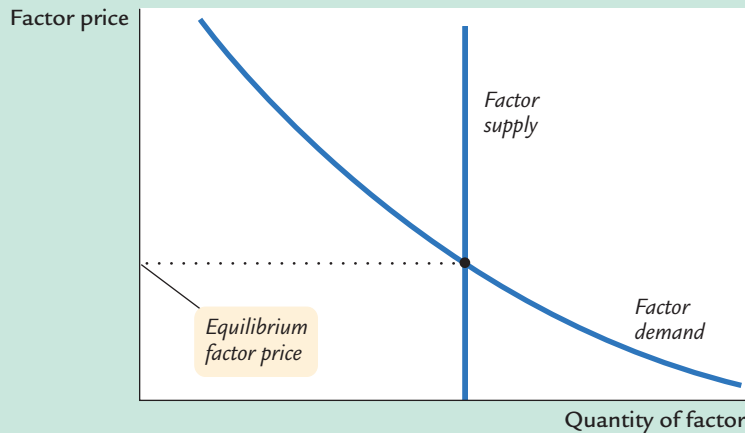
To understand factor prices and the distribution of income, we must examine the demand for the factors of production. Because factor demand arises from the thousands of firms that use capital and labor, we now look at the decisions faced by a typical firm about how much of these factors to employ.

The Decisions Facing the Competitive Firm

The simplest assumption to make about a typical firm is that it is **competitive**. A competitive firm is small relative to the markets in which it trades, so it has little influence on market prices. For example, our firm produces a good and sells it at the market price. Because many firms produce this good, our firm can sell as



figure 3-2



How a Factor of Production Is Compensated The price paid to any factor of production depends on the supply and demand for that factor's services. Because we have assumed that supply is fixed, the supply curve is vertical. The demand curve is downward sloping. The intersection of supply and demand determines the equilibrium factor price.

much as it wants without causing the price of the good to fall, or it can stop selling altogether without causing the price of the good to rise. Similarly, our firm cannot influence the wages of the workers it employs because many other local firms also employ workers. The firm has no reason to pay more than the market wage, and if it tried to pay less, its workers would take jobs elsewhere. Therefore, the competitive firm takes the prices of its output and its inputs as given.

To make its product, the firm needs two factors of production, capital and labor. As we did for the aggregate economy, we represent the firm's production technology by the production function

$$Y = F(K, L),$$

where Y is the number of units produced (the firm's output), K the number of machines used (the amount of capital), and L the number of hours worked by the firm's employees (the amount of labor). The firm produces more output if it has more machines or if its employees work more hours.

The firm sells its output at a price P , hires workers at a wage W , and rents capital at a rate R . Notice that when we speak of firms renting capital, we are assuming that households own the economy's stock of capital. In this analysis, households rent out their capital, just as they sell their labor. The firm obtains both factors of production from the households that own them.¹

The goal of the firm is to maximize profit. *Profit* is revenue minus costs—it is what the owners of the firm keep after paying for the costs of production. Revenue equals $P \times Y$, the selling price of the good P multiplied by the amount of

¹ This is a simplification. In the real world, the ownership of capital is indirect because firms own capital and households own the firms. That is, real firms have two functions: owning capital and producing output. To help us understand how the factors of production are compensated, however, we assume that firms only produce output and that households own capital directly.

the good the firm produces Y . Costs include both labor costs and capital costs. Labor costs equal $W \times L$, the wage W times the amount of labor L . Capital costs equal $R \times K$, the rental price of capital R times the amount of capital K . We can write

$$\begin{aligned}\text{Profit} &= \text{Revenue} - \text{Labor Costs} - \text{Capital Costs} \\ &= PY - WL - RK.\end{aligned}$$

To see how profit depends on the factors of production, we use the production function $Y = F(K, L)$ to substitute for Y to obtain

$$\text{Profit} = PF(K, L) - WL - RK.$$

This equation shows that profit depends on the product price P , the factor prices W and R , and the factor quantities L and K . The competitive firm takes the product price and the factor prices as given and chooses the amounts of labor and capital that maximize profit.

The Firm's Demand for Factors

We now know that our firm will hire labor and rent capital in the quantities that maximize profit. But what are those profit-maximizing quantities? To answer this question, we first consider the quantity of labor and then the quantity of capital.

The Marginal Product of Labor The more labor the firm employs, the more output it produces. The **marginal product of labor (MPL)** is the extra amount of output the firm gets from one extra unit of labor, holding the amount of capital fixed. We can express this using the production function:

$$MPL = F(K, L + 1) - F(K, L).$$

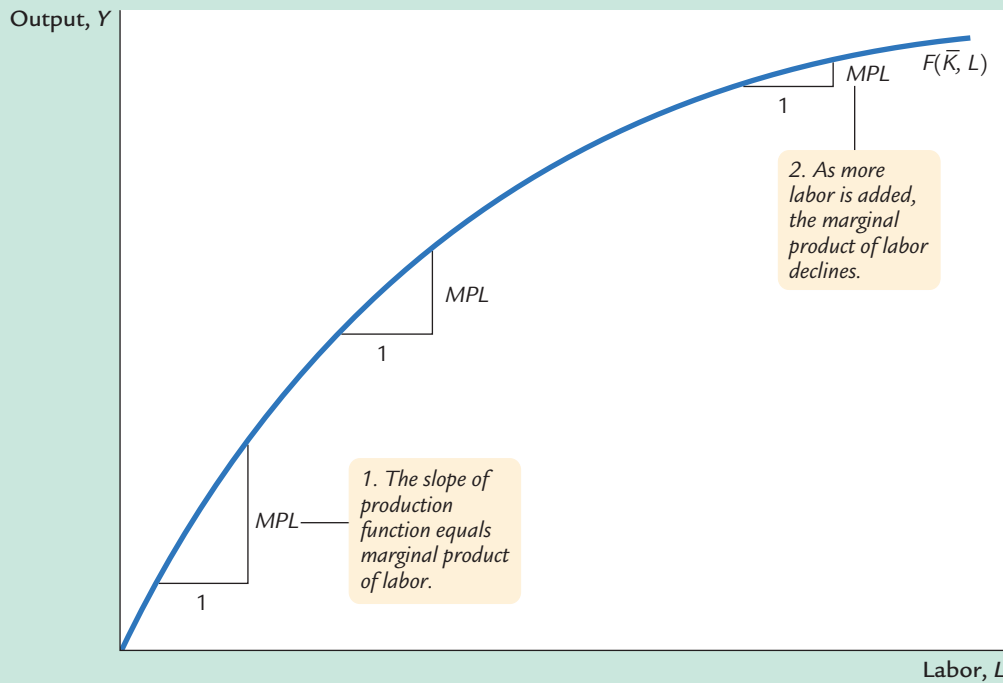
The first term on the right-hand side is the amount of output produced with K units of capital and $L + 1$ units of labor; the second term is the amount of output produced with K units of capital and L units of labor. This equation states that the marginal product of labor is the difference between the amount of output produced with $L + 1$ units of labor and the amount produced with only L units of labor.

Most production functions have the property of **diminishing marginal product**: holding the amount of capital fixed, the marginal product of labor decreases as the amount of labor increases. Consider again the production of bread at a bakery. As a bakery hires more labor, it produces more bread. The MPL is the amount of extra bread produced when an extra unit of labor is hired. As more labor is added to a fixed amount of capital, however, the MPL falls. Fewer additional loaves are produced because workers are less productive when the kitchen is more crowded. In other words, holding the size of the kitchen fixed, each additional worker adds fewer loaves of bread to the bakery's output.

Figure 3-3 graphs the production function. It illustrates what happens to the amount of output when we hold the amount of capital constant and vary the



figure 3-3



The Production Function This curve shows how output depends on labor input, holding the amount of capital constant. The marginal product of labor *MPL* is the change in output when the labor input is increased by 1 unit. As the amount of labor increases, the production function becomes flatter, indicating diminishing marginal product.

amount of labor. This figure shows that the marginal product of labor is the slope of the production function. As the amount of labor increases, the production function becomes flatter, indicating diminishing marginal product.

From the Marginal Product of Labor to Labor Demand When the competitive, profit-maximizing firm is deciding whether to hire an additional unit of labor, it considers how that decision would affect profits. It therefore compares the extra revenue from the increased production that results from the added labor to the extra cost of higher spending on wages. The increase in revenue from an additional unit of labor depends on two variables: the marginal product of labor and the price of the output. Because an extra unit of labor produces *MPL* units of output and each unit of output sells for *P* dollars, the extra revenue is $P \times MPL$. The extra cost of hiring one more unit of labor is the wage *W*. Thus, the change in profit from hiring an additional unit of labor is

$$\begin{aligned} \Delta \text{ Profit} &= \Delta \text{ Revenue} - \Delta \text{ Cost} \\ &= (P \times MPL) - W. \end{aligned}$$

The symbol Δ (called *delta*) denotes the change in a variable.

We can now answer the question we asked at the beginning of this section: How much labor does the firm hire? The firm's manager knows that if the extra revenue $P \times MPL$ exceeds the wage W , an extra unit of labor increases profit. Therefore, the manager continues to hire labor until the next unit would no longer be profitable—that is, until the MPL falls to the point where the extra revenue equals the wage. The firm's demand for labor is determined by

$$P \times MPL = W.$$

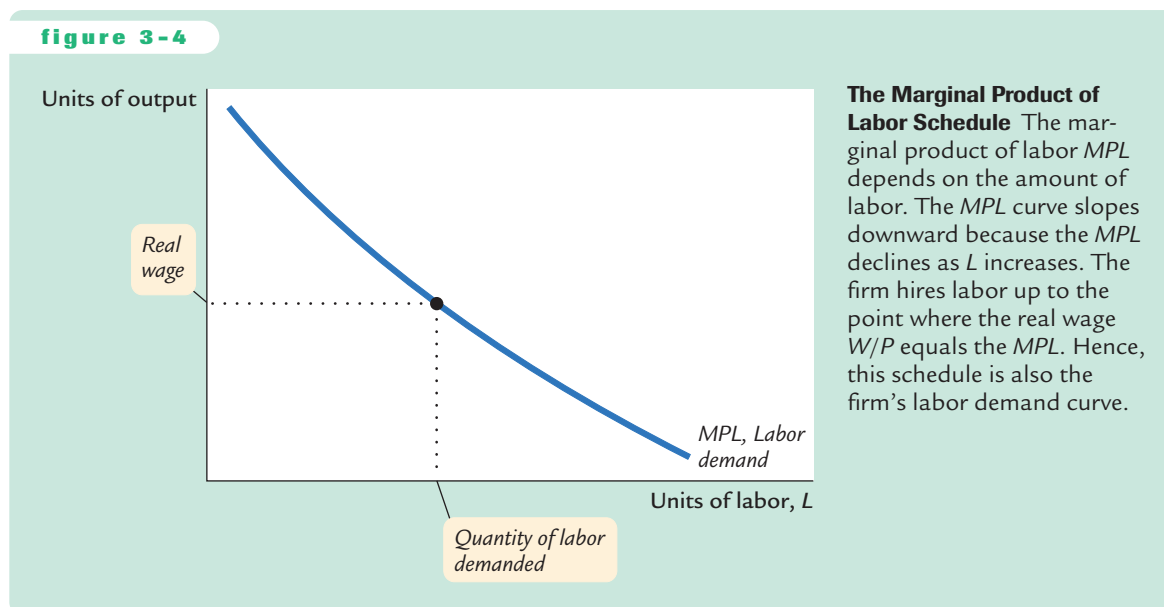
We can also write this as

$$MPL = W/P.$$

W/P is the **real wage**—the payment to labor measured in units of output rather than in dollars. To maximize profit, the firm hires up to the point at which the marginal product of labor equals the real wage.

For example, again consider a bakery. Suppose the price of bread P is \$2 per loaf, and a worker earns a wage W of \$20 per hour. The real wage W/P is 10 loaves per hour. In this example, the firm keeps hiring workers as long as each additional worker would produce at least 10 loaves per hour. When the MPL falls to 10 loaves per hour or less, hiring additional workers is no longer profitable.

Figure 3-4 shows how the marginal product of labor depends on the amount of labor employed (holding the firm's capital stock constant). That is, this figure graphs the MPL schedule. Because the MPL diminishes as the amount of labor increases, this curve slopes downward. For any given real wage, the firm hires up to the point at which the MPL equals the real wage. Hence, the MPL schedule is also the firm's labor demand curve.



The Marginal Product of Capital and Capital Demand The firm decides how much capital to rent in the same way it decides how much labor to hire. The **marginal product of capital (MPK)** is the amount of extra output the firm gets from an extra unit of capital, holding the amount of labor constant:

$$MPK = F(K + 1, L) - F(K, L).$$

Thus, the marginal product of capital is the difference between the amount of output produced with $K + 1$ units of capital and that produced with only K units of capital. Like labor, capital is subject to diminishing marginal product.

The increase in profit from renting an additional machine is the extra revenue from selling the output of that machine minus the machine's rental price:

$$\begin{aligned}\Delta\text{Profit} &= \Delta\text{Revenue} - \Delta\text{Cost} \\ &= (P \times MPK) - R.\end{aligned}$$

To maximize profit, the firm continues to rent more capital until the MPK falls to equal the real rental price:

$$MPK = R/P.$$

The **real rental price of capital** is the rental price measured in units of goods rather than in dollars.

To sum up, the competitive, profit-maximizing firm follows a simple rule about how much labor to hire and how much capital to rent. *The firm demands each factor of production until that factor's marginal product falls to equal its real factor price.*

The Division of National Income

Having analyzed how a firm decides how much of each factor to employ, we can now explain how the markets for the factors of production distribute the economy's total income. If all firms in the economy are competitive and profit maximizing, then each factor of production is paid its marginal contribution to the production process. The real wage paid to each worker equals the MPL , and the real rental price paid to each owner of capital equals the MPK . The total real wages paid to labor are therefore $MPL \times L$, and the total real return paid to capital owners is $MPK \times K$.

The income that remains after the firms have paid the factors of production is the **economic profit** of the owners of the firms. Real economic profit is

$$\text{Economic Profit} = Y - (MPL \times L) - (MPK \times K).$$

Because we want to examine the distribution of national income, we rearrange the terms as follows:

$$Y = (MPL \times L) + (MPK \times K) + \text{Economic Profit}.$$

Total income is divided among the return to labor, the return to capital, and economic profit.

How large is economic profit? The answer is surprising: if the production function has the property of constant returns to scale, as is often thought to be the case, then economic profit must be zero. That is, nothing is left after the factors of production are paid. This conclusion follows from a famous mathematical result called *Euler's theorem*,² which states that if the production function has constant returns to scale, then

$$F(K, L) = (MPK \times K) + (MPL \times L).$$

If each factor of production is paid its marginal product, then the sum of these factor payments equals total output. In other words, constant returns to scale, profit maximization, and competition together imply that economic profit is zero.

If economic profit is zero, how can we explain the existence of “profit” in the economy? The answer is that the term “profit” as normally used is different from economic profit. We have been assuming that there are three types of agents: workers, owners of capital, and owners of firms. Total income is divided among wages, return to capital, and economic profit. In the real world, however, most firms own rather than rent the capital they use. Because firm owners and capital owners are the same people, economic profit and the return to capital are often lumped together. If we call this alternative definition **accounting profit**, we can say that

$$\text{Accounting Profit} = \text{Economic Profit} + (MPK \times K).$$

Under our assumptions—constant returns to scale, profit maximization, and competition—economic profit is zero. If these assumptions approximately describe the world, then the “profit” in the national income accounts must be mostly the return to capital.

We can now answer the question posed at the beginning of this chapter about how the income of the economy is distributed from firms to households. Each factor of production is paid its marginal product, and these factor payments exhaust total output. *Total output is divided between the payments to capital and the payments to labor, depending on their marginal productivities.*

CASE STUDY

The Black Death and Factor Prices

As we have just learned, in the neoclassical theory of distribution, factor prices equal the marginal products of the factors of production. Because the marginal products depend on the quantities of the factors, a change in the quantity of any one factor alters the marginal products of all the factors. Therefore, a change in the supply of a factor alters equilibrium factor prices.

Fourteenth-century Europe provides a vivid example of how factor quantities affect factor prices. The outbreak of the bubonic plague—the Black Death—in

² *Mathematical note:* To prove Euler's theorem, begin with the definition of constant returns to scale: $zY = F(zK, zL)$. Now differentiate with respect to z and then evaluate at $z = 1$.

1348 reduced the population of Europe by about one-third within a few years. Because the marginal product of labor increases as the amount of labor falls, this massive reduction in the labor force raised the marginal product of labor. (The economy moved to the left along the curves in Figures 3-3 and 3-4.) Real wages did increase substantially during the plague years—doubling, by some estimates. The peasants who were fortunate enough to survive the plague enjoyed economic prosperity.

The reduction in the labor force caused by the plague also affected the return to land, the other major factor of production in medieval Europe. With fewer workers available to farm the land, an additional unit of land produced less additional output. This fall in the marginal product of land led to a decline in real rents of 50 percent or more. Thus, while the peasant classes prospered, the landed classes suffered reduced incomes.³

3-3 What Determines the Demand for Goods and Services?

We have seen what determines the level of production and how the income from production is distributed to workers and owners of capital. We now continue our tour of the circular flow diagram, Figure 3-1, and examine how the output from production is used.

In Chapter 2 we identified the four components of GDP:

- Consumption (C)
- Investment (I)
- Government purchases (G)
- Net exports (NX).

The circular flow diagram contains only the first three components. For now, to simplify the analysis, we assume a *closed economy*—a country that does not trade with other countries. Thus, net exports are always zero. (We examine the macroeconomics of *open economies* in Chapter 5.)

A closed economy has three uses for the goods and services it produces. These three components of GDP are expressed in the national income accounts identity:

$$Y = C + I + G.$$

Households consume some of the economy's output; firms and households use some of the output for investment; and the government buys some of the output for public purposes. We want to see how GDP is allocated among these three uses.

³ Carlo M. Cipolla, *Before the Industrial Revolution: European Society and Economy, 1000–1700*, 2d ed. (New York: Norton, 1980), 200–202.

Consumption

When we eat food, wear clothing, or go to a movie, we are consuming some of the output of the economy. All forms of consumption together make up two-thirds of GDP. Because consumption is so large, macroeconomists have devoted much energy to studying how households decide how much to consume. Chapter 16 examines this work in detail. Here we consider the simplest story of consumer behavior.

Households receive income from their labor and their ownership of capital, pay taxes to the government, and then decide how much of their after-tax income to consume and how much to save. As we discussed in Section 3-2, the income that households receive equals the output of the economy Y . The government then taxes households an amount T . (Although the government imposes many kinds of taxes, such as personal and corporate income taxes and sales taxes, for our purposes we can lump all these taxes together.) We define income after the payment of all taxes, $Y - T$, as **disposable income**. Households divide their disposable income between consumption and saving.

We assume that the level of consumption depends directly on the level of disposable income. The higher the disposable income, the greater the consumption. Thus,

$$C = C(Y - T).$$

This equation states that consumption is a function of disposable income. The relationship between consumption and disposable income is called the **consumption function**.

The **marginal propensity to consume (MPC)** is the amount by which consumption changes when disposable income increases by one dollar. The *MPC* is between zero and one: an extra dollar of income increases consumption, but by less than one dollar. Thus, if households obtain an extra dollar of income, they save a portion of it. For example, if the *MPC* is 0.7, then households spend 70 cents of each additional dollar of disposable income on consumer goods and services and save 30 cents.

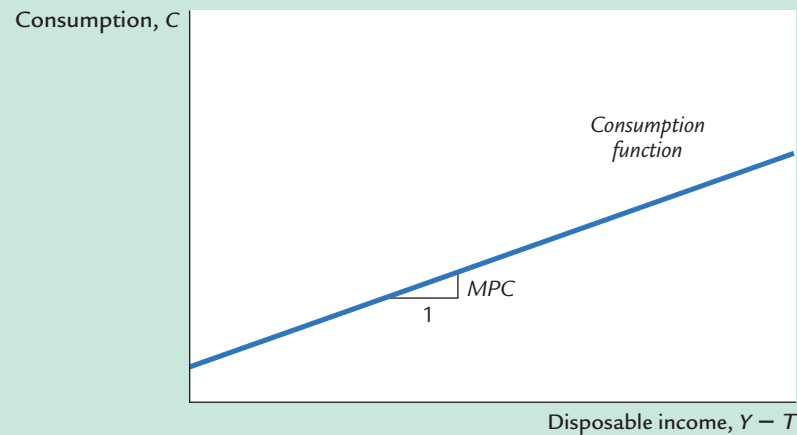
Figure 3-5 illustrates the consumption function. The slope of the consumption function tells us how much consumption increases when disposable income increases by one dollar. That is, the slope of the consumption function is the *MPC*.

Investment

Both firms and households purchase investment goods. Firms buy investment goods to add to their stock of capital and to replace existing capital as it wears out. Households buy new houses, which are also part of investment. Total investment in the United States averages about 15 percent of GDP.

The quantity of investment goods demanded depends on the interest rate, which measures the cost of the funds used to finance investment. For an investment project to be profitable, its return (the revenue from increased future

figure 3-5

**The Consumption Function**

The consumption function relates consumption C to disposable income $Y - T$. The marginal propensity to consume MPC is the amount by which consumption increases when disposable income increases by one dollar.

production of goods and services) must exceed its cost (the payments for borrowed funds). If the interest rate rises, fewer investment projects are profitable, and the quantity of investment goods demanded falls.

For example, suppose a firm is considering whether it should build a \$1 million factory that would yield a return of \$100,000 per year, or 10 percent. The firm compares this return to the cost of borrowing the \$1 million. If the interest rate is below 10 percent, the firm borrows the money in financial markets and makes the investment. If the interest rate is above 10 percent, the firm forgoes the investment opportunity and does not build the factory.

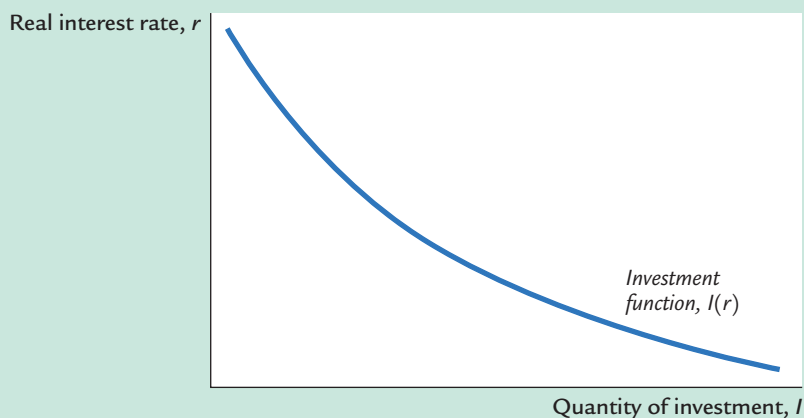
The firm makes the same investment decision even if it does not have to borrow the \$1 million but rather uses its own funds. The firm can always deposit this money in a bank or a money market fund and earn interest on it. Building the factory is more profitable than the deposit if and only if the interest rate is less than the 10 percent return on the factory.

A person wanting to buy a new house faces a similar decision. The higher the interest rate, the greater the cost of carrying a mortgage. A \$100,000 mortgage costs \$8,000 per year if the interest rate is 8 percent and \$10,000 per year if the interest rate is 10 percent. As the interest rate rises, the cost of owning a home rises, and the demand for new homes falls.

When studying the role of interest rates in the economy, economists distinguish between the nominal interest rate and the real interest rate. This distinction is relevant when the overall level of prices is changing. The **nominal interest rate** is the interest rate as usually reported: it is the rate of interest that investors pay to borrow money. The **real interest rate** is the nominal interest rate corrected for the effects of inflation. If the nominal interest rate is 8 percent and the inflation rate is 3 percent, then the real interest rate is 5 percent. In Chapter 4 we discuss the relation between nominal and real interest rates in detail. Here it is sufficient to note that the real interest rate measures the true cost of borrowing and, thus, determines the quantity of investment.



figure 3-6

**The Investment Function**

The investment function relates the quantity of investment I to the real interest rate r . Investment depends on the real interest rate because the interest rate is the cost of borrowing. The investment function slopes downward: when the interest rate rises, fewer investment projects are profitable.

We can summarize this discussion with an equation relating investment I to the real interest rate r :

$$I = I(r).$$

Figure 3-6 shows this investment function. It slopes downward, because as the interest rate rises, the quantity of investment demanded falls.

Government Purchases

Government purchases are the third component of the demand for goods and services. The federal government buys guns, missiles, and the services of government employees. Local governments buy library books, build schools, and hire teachers. Governments at all levels build roads and other public works. All these transactions make up government purchases of goods and services, which account for about 20 percent of GDP in the United States.

These purchases are only one type of government spending. The other type is transfer payments to households, such as welfare for the poor and Social Security payments for the elderly. Unlike government purchases, transfer payments are not made in exchange for some of the economy's output of goods and services. Therefore, they are not included in the variable G .

Transfer payments do affect the demand for goods and services indirectly. Transfer payments are the opposite of taxes: they increase households' disposable income, just as taxes reduce disposable income. Thus, an increase in transfer payments financed by an increase in taxes leaves disposable income unchanged. We can now revise our definition of T to equal taxes minus transfer payments. Disposable income, $Y - T$, includes both the negative impact of taxes and the positive impact of transfer payments.

If government purchases equal taxes minus transfers, then $G = T$, and the government has a *balanced budget*. If G exceeds T , the government runs a *budget*

FYI The Many Different Interest Rates

If you look in the business section of a newspaper, you will find many different interest rates reported. By contrast, throughout this book, we will talk about “the” interest rate, as if there were only one interest rate in the economy. The only distinction we will make is between the nominal interest rate (which is not corrected for inflation) and the real interest rate (which is corrected for inflation). Almost all of the interest rates reported in the newspaper are nominal.

Why does the newspaper report so many interest rates? The various interest rates differ in three ways:

- ▶ **Term.** Some loans in the economy are for short periods of time, even as short as overnight. Other loans are for 30 years or even longer. The interest rate on a loan depends on its term. Long-term interest rates are usually, but not always, higher than short-term interest rates.
- ▶ **Credit risk.** In deciding whether to make a loan, a lender must take into account the probability that the borrower will repay. The law allows borrowers to default on their loans by declaring bankruptcy. The higher the perceived prob-

ability of default, the higher the interest rate. The safest credit risk is the government, and so government bonds tend to pay a low interest rate. At the other extreme, financially shaky corporations can raise funds only by issuing *junk bonds*, which pay a high interest rate to compensate for the high risk of default.

- ▶ **Tax treatment.** The interest on different types of bonds is taxed differently. Most important, when state and local governments issue bonds, called *municipal bonds*, the holders of the bonds do not pay federal income tax on the interest income. Because of this tax advantage, municipal bonds pay a lower interest rate.

When you see two different interest rates in the newspaper, you can almost always explain the difference by considering the term, the credit risk, and the tax treatment of the loan.

Although there are many different interest rates in the economy, macroeconomists can usually ignore these distinctions. The various interest rates tend to move up and down together. The assumption that there is only one interest rate is, for our purposes, a useful simplification.

deficit, which it funds by issuing government debt—that is, by borrowing in the financial markets. If G is less than T , the government runs a *budget surplus*, which it can use to repay some of its outstanding debt.

Here we do not try to explain the political process that leads to a particular fiscal policy—that is, to the level of government purchases and taxes. Instead, we take government purchases and taxes as exogenous variables. To denote that these variables are fixed outside of our model of national income, we write

$$G = \bar{G}.$$

$$T = \bar{T}.$$

We do, however, want to examine the impact of fiscal policy on the variables determined within the model, the endogenous variables. The endogenous variables here are consumption, investment, and the interest rate.

To see how the exogenous variables affect the endogenous variables, we must complete the model. This is the subject of the next section.

3-4 What Brings the Supply and Demand for Goods and Services Into Equilibrium?

We have now come full circle in the circular flow diagram, Figure 3-1. We began by examining the supply of goods and services, and we have just discussed the demand for them. How can we be certain that all these flows balance? In other words, what ensures that the sum of consumption, investment, and government purchases equals the amount of output produced? We will see that in this classical model, the interest rate has the crucial role of equilibrating supply and demand.

There are two ways to think about the role of the interest rate in the economy. We can consider how the interest rate affects the supply and demand for goods or services. Or we can consider how the interest rate affects the supply and demand for loanable funds. As we will see, these two approaches are two sides of the same coin.

Equilibrium in the Market for Goods and Services: The Supply and Demand for the Economy's Output

The following equations summarize the discussion of the demand for goods and services in Section 3-3:

$$Y = C + I + G.$$

$$C = C(Y - T).$$

$$I = I(r).$$

$$G = \bar{G}.$$

$$T = \bar{T}.$$

The demand for the economy's output comes from consumption, investment, and government purchases. Consumption depends on disposable income; investment depends on the real interest rate; and government purchases and taxes are the exogenous variables set by fiscal policymakers.

To this analysis, let's add what we learned about the supply of goods and services in Section 3-1. There we saw that the factors of production and the production function determine the quantity of output supplied to the economy:

$$\begin{aligned} Y &= F(\bar{K}, \bar{L}) \\ &= \bar{Y}. \end{aligned}$$

Now let's combine these equations describing the supply and demand for output. If we substitute the consumption function and the investment function into the national income accounts identity, we obtain

$$Y = C(Y - T) + I(r) + G.$$

Because the variables G and T are fixed by policy, and the level of output Y is fixed by the factors of production and the production function, we can write

$$\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}.$$

This equation states that the supply of output equals its demand, which is the sum of consumption, investment, and government purchases.

Notice that the interest rate r is the only variable not already determined in the last equation. This is because the interest rate still has a key role to play: it must adjust to ensure that the demand for goods equals the supply. The greater the interest rate, the lower the level of investment, and thus the lower the demand for goods and services, $C + I + G$. If the interest rate is too high, investment is too low, and the demand for output falls short of the supply. If the interest rate is too low, investment is too high, and the demand exceeds the supply. *At the equilibrium interest rate, the demand for goods and services equals the supply.*

This conclusion may seem somewhat mysterious. One might wonder how the interest rate gets to the level that balances the supply and demand for goods and services. The best way to answer this question is to consider how financial markets fit into the story.

Equilibrium in the Financial Markets: The Supply and Demand for Loanable Funds

Because the interest rate is the cost of borrowing and the return to lending in financial markets, we can better understand the role of the interest rate in the economy by thinking about the financial markets. To do this, rewrite the national income accounts identity as

$$Y - C - G = I.$$

The term $Y - C - G$ is the output that remains after the demands of consumers and the government have been satisfied; it is called **national saving** or simply **saving** (S). In this form, the national income accounts identity shows that saving equals investment.

To understand this identity more fully, we can split national saving into two parts—one part representing the saving of the private sector and the other representing the saving of the government:

$$(Y - T - C) + (T - G) = I.$$

The term $(Y - T - C)$ is disposable income minus consumption, which is **private saving**. The term $(T - G)$ is government revenue minus government spending, which is **public saving**. (If government spending exceeds government revenue, the government runs a budget deficit, and public saving is negative.) National saving is the sum of private and public saving. The circular flow

diagram in Figure 3-1 reveals an interpretation of this equation: this equation states that the flows into the financial markets (private and public saving) must balance the flows out of the financial markets (investment).

To see how the interest rate brings financial markets into equilibrium, substitute the consumption function and the investment function into the national income accounts identity:

$$Y - C(Y - T) - G = I(r).$$

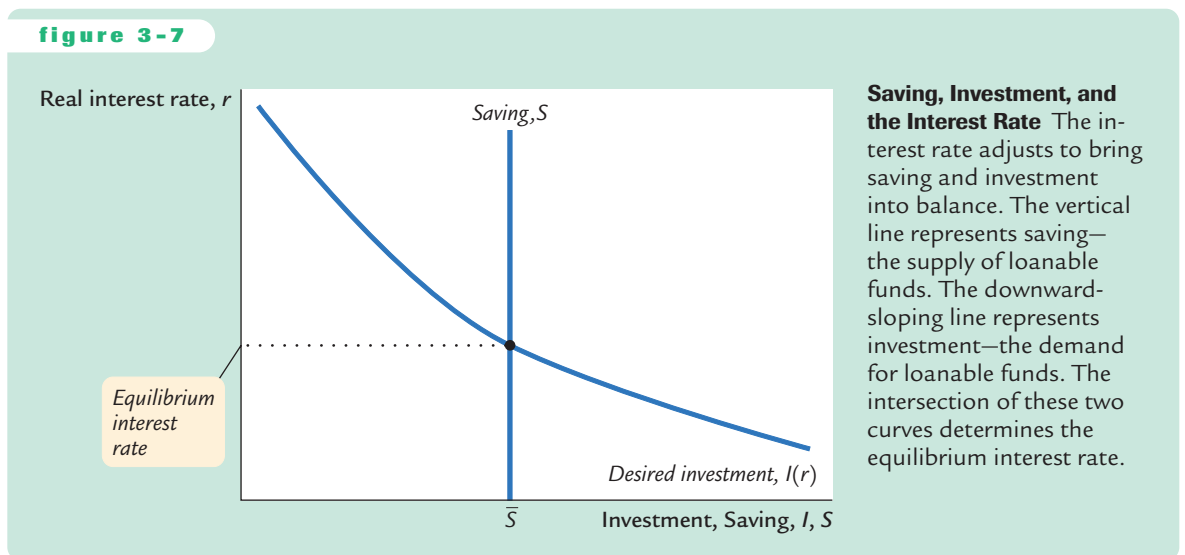
Next, note that G and T are fixed by policy and Y is fixed by the factors of production and the production function:

$$\begin{aligned} \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G} &= I(r) \\ \bar{S} &= I(r). \end{aligned}$$

The left-hand side of this equation shows that national saving depends on income Y and the fiscal-policy variables G and T . For fixed values of Y , G , and T , national saving S is also fixed. The right-hand side of the equation shows that investment depends on the interest rate.

Figure 3-7 graphs saving and investment as a function of the interest rate. The saving function is a vertical line because in this model saving does not depend on the interest rate (although we relax this assumption later). The investment function slopes downward: the higher the interest rate, the fewer profitable investment projects.

From a quick glance at Figure 3-7, one might think it was a supply-and-demand diagram for a particular good. In fact, saving and investment can be interpreted in terms of supply and demand. In this case, the “good” is **loanable funds**, and its “price” is the interest rate. Saving is the supply of loanable funds—



households lend their saving to investors or deposit their saving in a bank that then loans the funds out. Investment is the demand for loanable funds—investors borrow from the public directly by selling bonds or indirectly by borrowing from banks. Because investment depends on the interest rate, the quantity of loanable funds demanded also depends on the interest rate.

The interest rate adjusts until the amount that firms want to invest equals the amount that households want to save. If the interest rate is too low, investors want more of the economy's output than households want to save. Equivalently, the quantity of loanable funds demanded exceeds the quantity supplied. When this happens, the interest rate rises. Conversely, if the interest rate is too high, households want to save more than firms want to invest; because the quantity of loanable funds supplied is greater than the quantity demanded, the interest rate falls. The equilibrium interest rate is found where the two curves cross. *At the equilibrium interest rate, households' desire to save balances firms' desire to invest, and the quantity of loanable funds supplied equals the quantity demanded.*

Changes in Saving: The Effects of Fiscal Policy

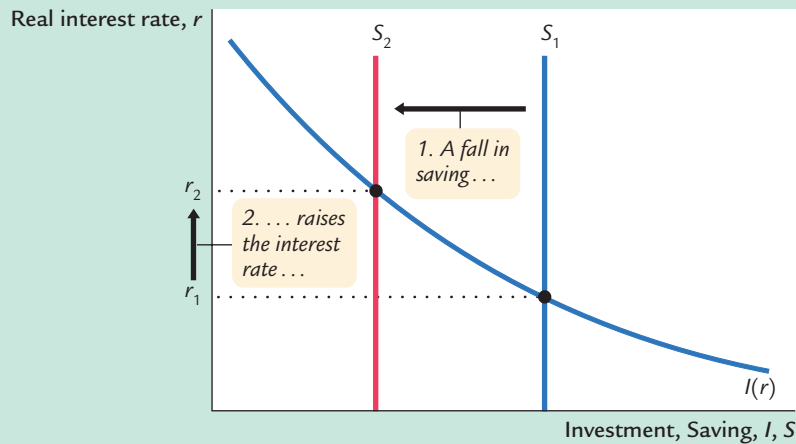
We can use our model to show how fiscal policy affects the economy. When the government changes its spending or the level of taxes, it affects the demand for the economy's output of goods and services and alters national saving, investment, and the equilibrium interest rate.

An Increase in Government Purchases Consider first the effects of an increase in government purchases of an amount ΔG . The immediate impact is to increase the demand for goods and services by ΔG . But since total output is fixed by the factors of production, the increase in government purchases must be met by a decrease in some other category of demand. Because disposable income $Y - T$ is unchanged, consumption C is unchanged. The increase in government purchases must be met by an equal decrease in investment.

To induce investment to fall, the interest rate must rise. Hence, the increase in government purchases causes the interest rate to increase and investment to decrease. Government purchases are said to **crowd out** investment.

To grasp the effects of an increase in government purchases, consider the impact on the market for loanable funds. Because the increase in government purchases is not accompanied by an increase in taxes, the government finances the additional spending by borrowing—that is, by reducing public saving. With private saving unchanged, this government borrowing reduces national saving. As Figure 3-8 shows, a reduction in national saving is represented by a leftward shift in the supply of loanable funds available for investment. At the initial interest rate, the demand for loanable funds exceeds the supply. The equilibrium interest rate rises to the point where the investment schedule crosses the new saving schedule. Thus, an increase in government purchases causes the interest rate to rise from r_1 to r_2 .

figure 3-8



A Reduction in Saving A reduction in saving, possibly the result of a change in fiscal policy, shifts the saving schedule to the left. The new equilibrium is the point at which the new saving schedule crosses the investment schedule. A reduction in saving lowers the amount of investment and raises the interest rate. Fiscal-policy actions that reduce saving are said to crowd out investment.

CASE STUDY

Wars and Interest Rates in the United Kingdom, 1730–1920

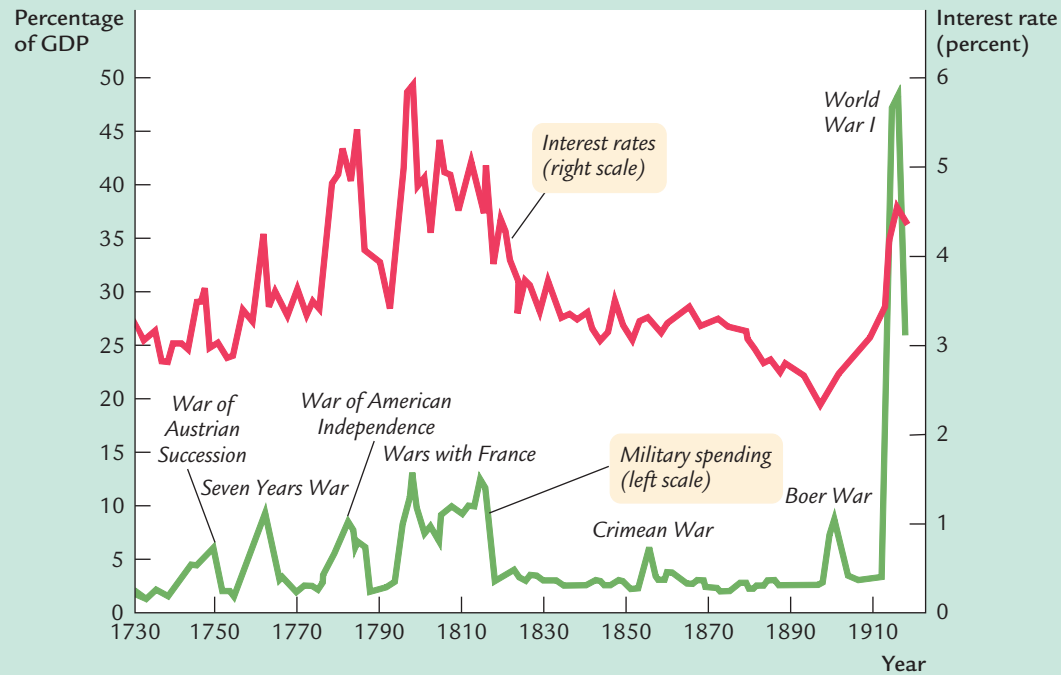
Wars are traumatic—both for those who fight them and for a nation’s economy. Because the economic changes accompanying them are often large, wars provide a natural experiment with which economists can test their theories. We can learn about the economy by seeing how in wartime the endogenous variables respond to the major changes in the exogenous variables.

One exogenous variable that changes substantially in wartime is the level of government purchases. Figure 3-9 shows military spending as a percentage of GDP for the United Kingdom from 1730 to 1919. This graph shows, as one would expect, that government purchases rose suddenly and dramatically during the eight wars of this period.

Our model predicts that this wartime increase in government purchases—and the increase in government borrowing to finance the wars—should have raised the demand for goods and services, reduced the supply of loanable funds, and raised the interest rate. To test this prediction, Figure 3-9 also shows the interest rate on long-term government bonds, called *consols* in the United Kingdom. A positive association between military purchases and interest rates is apparent in this figure. These data support the model’s prediction: interest rates do tend to rise when government purchases increase.⁴

⁴ Daniel K. Benjamin and Levis A. Kochin, “War, Prices, and Interest Rates: A Martial Solution to Gibson’s Paradox,” in M. D. Bordo and A. J. Schwartz, eds., *A Retrospective on the Classical Gold Standard, 1821–1931* (Chicago: University of Chicago Press, 1984), 587–612; Robert J. Barro, “Government Spending, Interest Rates, Prices, and Budget Deficits in the United Kingdom, 1701–1918,” *Journal of Monetary Economics* 20 (September 1987): 221–248.

figure 3-9



Military Spending and the Interest Rate in the United Kingdom This figure shows military spending as a percentage of GDP in the United Kingdom from 1730 to 1919. Not surprisingly, military spending rose substantially during each of the eight wars of this period. This figure also shows that the interest rate tended to rise when military spending rose.

Source: Series constructed from various sources described in Robert J. Barro, "Government Spending, Interest Rates, Prices, and Budget Deficits in the United Kingdom, 1701-1918," *Journal of Monetary Economics* 20 (September 1987): 221-248.

One problem with using wars to test theories is that many economic changes may be occurring at the same time. For example, in World War II, while government purchases increased dramatically, rationing also restricted consumption of many goods. In addition, the risk of defeat in the war and default by the government on its debt presumably increases the interest rate the government must pay. Economic models predict what happens when one exogenous variable changes and all the other exogenous variables remain constant. In the real world, however, many exogenous variables may change at once. Unlike controlled laboratory experiments, the natural experiments on which economists must rely are not always easy to interpret.

A Decrease in Taxes Now consider a reduction in taxes of ΔT . The immediate impact of the tax cut is to raise disposable income and thus to raise consumption. Disposable income rises by ΔT , and consumption rises by an amount equal to



ΔT times the marginal propensity to consume MPC . The higher the MPC , the greater the impact of the tax cut on consumption.

Because the economy's output is fixed by the factors of production and the level of government purchases is fixed by the government, the increase in consumption must be met by a decrease in investment. For investment to fall, the interest rate must rise. Hence, a reduction in taxes, like an increase in government purchases, crowds out investment and raises the interest rate.

We can also analyze the effect of a tax cut by looking at saving and investment. Because the tax cut raises disposable income by ΔT , consumption goes up by $MPC \times \Delta T$. National saving S , which equals $Y - C - G$, falls by the same amount as consumption rises. As in Figure 3-8, the reduction in saving shifts the supply of loanable funds to the left, which increases the equilibrium interest rate and crowds out investment.

Changes in Investment Demand

So far, we have discussed how fiscal policy can change national saving. We can also use our model to examine the other side of the market—the demand for investment. In this section we look at the causes and effects of changes in investment demand.

One reason investment demand might increase is technological innovation. Suppose, for example, that someone invents a new technology, such as the railroad or the computer. Before a firm or household can take advantage of the innovation, it must buy investment goods. The invention of the railroad had no value until railroad cars were produced and tracks were laid. The idea of the computer was not productive until computers were manufactured. Thus, technological innovation leads to an increase in investment demand.

Investment demand may also change because the government encourages or discourages investment through the tax laws. For example, suppose that the government increases personal income taxes and uses the extra revenue to provide tax cuts for those who invest in new capital. Such a change in the tax laws makes more investment projects profitable and, like a technological innovation, increases the demand for investment goods.

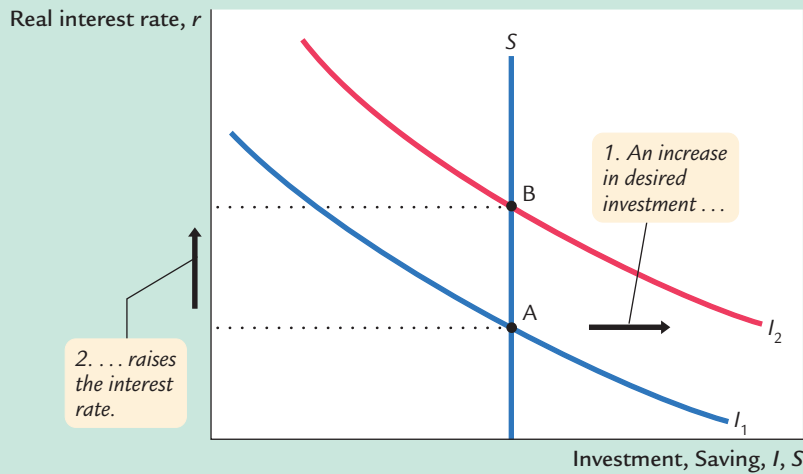
Figure 3-10 shows the effects of an increase in investment demand. At any given interest rate, the demand for investment goods (and also for loanable funds) is higher. This increase in demand is represented by a shift in the investment schedule to the right. The economy moves from the old equilibrium, point A, to the new equilibrium, point B.

The surprising implication of Figure 3-10 is that the equilibrium amount of investment is unchanged. Under our assumptions, the fixed level of saving determines the amount of investment; in other words, there is a fixed supply of loanable funds. An increase in investment demand merely raises the equilibrium interest rate.

We would reach a different conclusion, however, if we modified our simple consumption function and allowed consumption (and its flip side, saving) to



figure 3-10



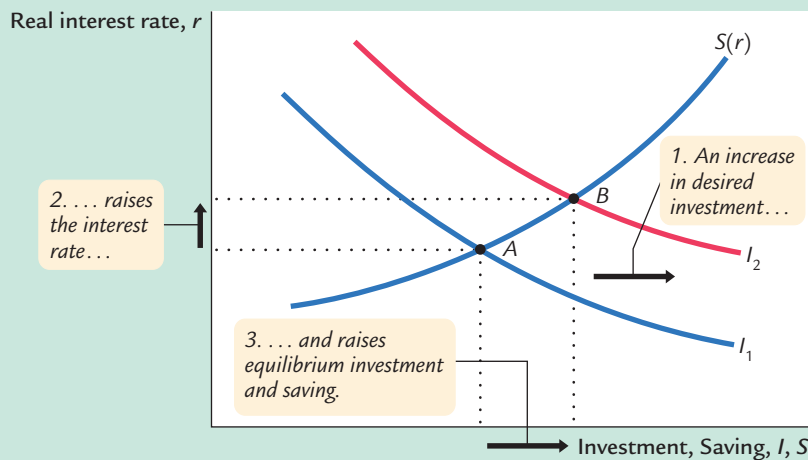
An Increase in the Demand for Investment

An increase in the demand for investment goods shifts the investment schedule to the right. At any given interest rate, the amount of investment is greater. The equilibrium moves from point A to point B. Because the amount of saving is fixed, the increase in investment demand raises the interest rate while leaving the equilibrium amount of investment unchanged.

depend on the interest rate. Because the interest rate is the return to saving (as well as the cost of borrowing), a higher interest rate might reduce consumption and increase saving. If so, the saving schedule would be upward sloping, rather than vertical.

With an upward-sloping saving schedule, an increase in investment demand would raise both the equilibrium interest rate and the equilibrium quantity of investment. Figure 3-11 shows such a change. The increase in the interest rate causes households to consume less and save more. The decrease in consumption frees resources for investment.

figure 3-11



An Increase in Investment Demand When Saving Depends on the Interest Rate

When saving is positively related to the interest rate, a rightward shift in the investment schedule increases the interest rate and the amount of investment. The higher interest rate induces people to increase saving, which in turn allows investment to increase.

FYI The Identification Problem

In our model, investment depends on the interest rate. The higher the interest rate, the fewer investment projects there are that are profitable. The investment schedule therefore slopes downward.

Economists who look at macroeconomic data, however, usually fail to find an obvious association between investment and interest rates. In years when interest rates are high, investment is not always low. In years when interest rates are low, investment is not always high.

How do we interpret this finding? Does it mean that investment does not depend on the interest rate? Does it suggest that our model of saving, investment, and the interest rate is inconsistent with how the economy actually functions?

Luckily, we do not have to discard our model. The inability to find an empirical relationship between investment and interest rates is an example of the *identification problem*. The identification problem arises when variables are related in more than one way. When we look at data, we are observing a combination of these different relationships, and it is difficult to “identify” any one of them.

To understand this problem more concretely, consider the relationships among saving, investment, and the interest rate. Suppose, on the one hand, that all changes in the interest rate resulted from changes in saving—that is, from

shifts in the saving schedule. Then, as shown in the left-hand side of panel (a) in Figure 3-12, all changes would represent movement along a fixed investment schedule. As the right-hand side of panel (a) shows, the data would trace out this investment schedule. Thus, we would observe a negative relationship between investment and interest rates.

Suppose, on the other hand, that all changes in the interest rate resulted from technological innovations—that is, from shifts in the investment schedule. Then, as shown in panel (b), all changes would represent movements in the investment schedule along a fixed saving schedule. As the right-hand side of panel (b) shows, the data would reflect this saving schedule. Thus, we would observe a positive relationship between investment and interest rates.

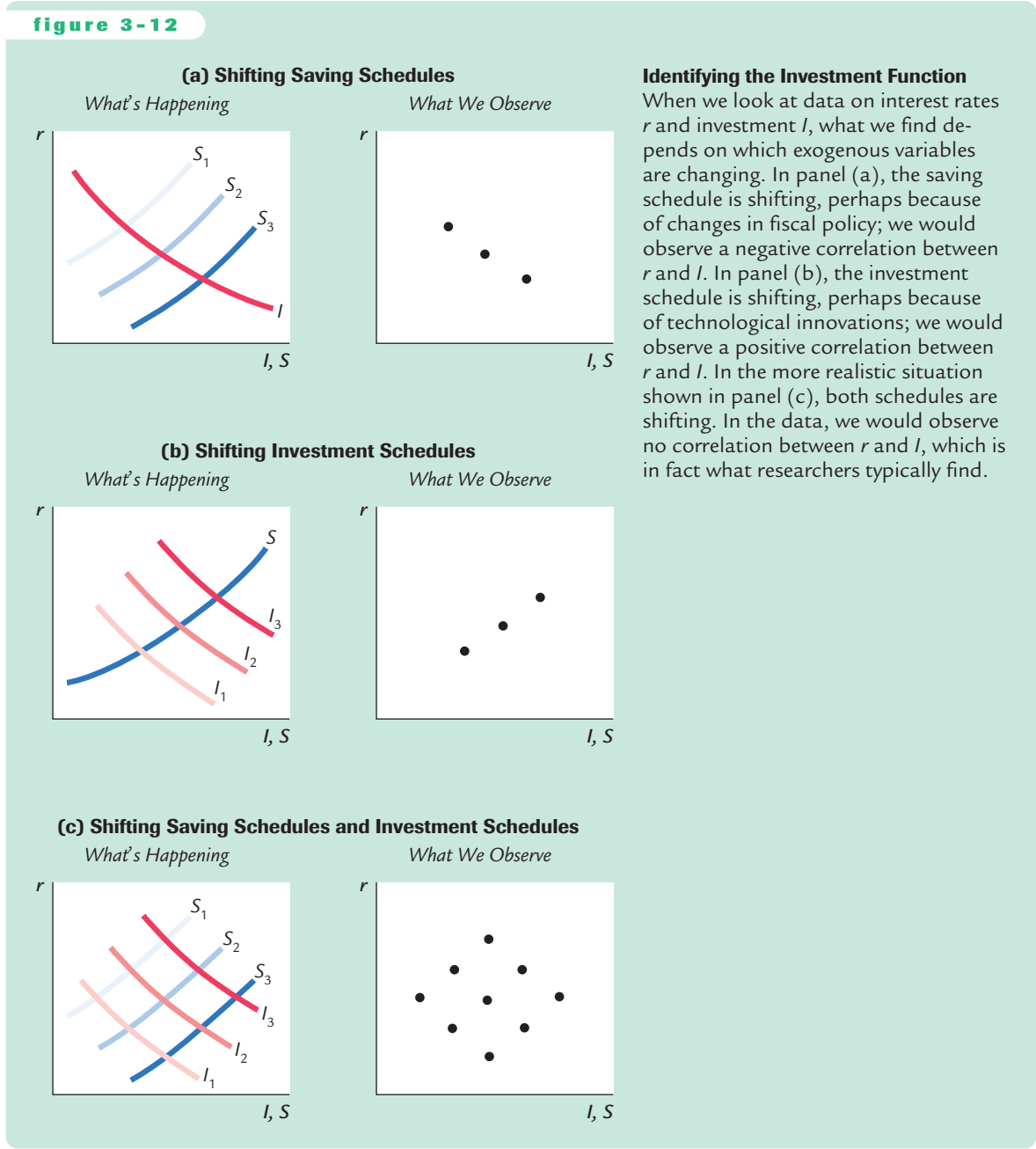
In the real world, interest rates change sometimes because of shifts in the saving schedule and sometimes because of shifts in the investment schedule. In this mixed case, as shown in panel (c), a plot of the data would reveal no recognizable relation between interest rates and the quantity of investment, just as economists observe in actual data. The moral of the story is simple and is applicable to many other situations: the empirical relationship we expect to observe depends crucially on which exogenous variables we think are changing.

3-5 Conclusion

In this chapter we have developed a model that explains the production, distribution, and allocation of the economy’s output of goods and services. Because the model incorporates all the interactions illustrated in the circular flow diagram in Figure 3-1, it is sometimes called a *general equilibrium model*. The model emphasizes how prices adjust to equilibrate supply and demand. Factor prices equilibrate factor markets. The interest rate equilibrates the supply and demand for goods and services (or, equivalently, the supply and demand for loanable funds).



figure 3-12



Throughout the chapter, we have discussed various applications of the model. The model can explain how income is divided among the factors of production and how factor prices depend on factor supplies. We have also used the model to discuss how fiscal policy alters the allocation of output among its alternative uses—consumption, investment, and government purchases—and how it affects the equilibrium interest rate.

At this point it is useful to review some of the simplifying assumptions we have made in this chapter. In the following chapters we relax some of these assumptions in order to address a greater range of questions.

- ▶ We have ignored the role of money, the asset with which goods and services are bought and sold. In Chapter 4 we discuss how money affects the economy and the influence of monetary policy.
- ▶ We have assumed that there is no trade with other countries. In Chapter 5 we consider how international interactions affect our conclusions.
- ▶ We have assumed that the labor force is fully employed. In Chapter 6 we examine the reasons for unemployment and see how public policy influences the level of unemployment.
- ▶ We have assumed that the capital stock, the labor force, and the production technology are fixed. In Chapters 7 and 8 we see how changes over time in each of these lead to growth in the economy's output of goods and services.
- ▶ We have ignored the role of short-run sticky prices. In Chapters 9 through 13, we develop a model of short-run fluctuations that includes sticky prices. We then discuss how the model of short-run fluctuations relates to the model of national income developed in this chapter.

Before going on to these chapters, go back to the beginning of this one and make sure you can answer the four groups of questions about national income that begin the chapter.

Summary

1. The factors of production and the production technology determine the economy's output of goods and services. An increase in one of the factors of production or a technological advance raises output.
2. Competitive, profit-maximizing firms hire labor until the marginal product of labor equals the real wage. Similarly, these firms rent capital until the marginal product of capital equals the real rental price. Therefore, each factor of production is paid its marginal product. If the production function has constant returns to scale, all output is used to compensate the inputs.
3. The economy's output is used for consumption, investment, and government purchases. Consumption depends positively on disposable income. Investment depends negatively on the real interest rate. Government purchases and taxes are the exogenous variables of fiscal policy.
4. The real interest rate adjusts to equilibrate the supply and demand for the economy's output—or, equivalently, to equilibrate the supply of loanable funds (saving) and the demand for loanable funds (investment). A decrease in national saving, perhaps because of an increase in government purchases or a



decrease in taxes, reduces the equilibrium amount of investment and raises the interest rate. An increase in investment demand, perhaps because of a technological innovation or a tax incentive for investment, also raises the interest rate. An increase in investment demand increases the quantity of investment only if higher interest rates stimulate additional saving.

KEY CONCEPTS

Factors of production	Marginal product of capital (<i>MPK</i>)	Nominal interest rate
Production function	Real rental price of capital	Real interest rate
Constant returns to scale	Economic profit versus accounting profit	National saving (saving)
Factor prices	Disposable income	Private saving
Competition	Consumption function	Public saving
Marginal product of labor (<i>MPL</i>)	Marginal propensity to consume (<i>MPC</i>)	Loanable funds
Diminishing marginal product		Crowding out
Real wage		

QUESTIONS FOR REVIEW

1. What determines the amount of output an economy produces?
2. Explain how a competitive, profit-maximizing firm decides how much of each factor of production to demand.
3. What is the role of constant returns to scale in the distribution of income?
4. What determines consumption and investment?
5. Explain the difference between government purchases and transfer payments. Give two examples of each.
6. What makes the demand for the economy's output of goods and services equal the supply?
7. Explain what happens to consumption, investment, and the interest rate when the government increases taxes.

PROBLEMS AND APPLICATIONS

1. Use the neoclassical theory of distribution to predict the impact on the real wage and the real rental price of capital of each of the following events:
 - a. A wave of immigration increases the labor force.
 - b. An earthquake destroys some of the capital stock.
 - c. A technological advance improves the production function.
2. If a 10-percent increase in both capital and labor causes output to increase by less than 10 percent, the production function is said to exhibit *decreasing returns to scale*. If it causes output to increase by more than 10 percent, the production function is said to exhibit *increasing returns to scale*. Why might a production function exhibit decreasing or increasing returns to scale?
3. According to the neoclassical theory of distribution, the real wage earned by any worker equals that worker's marginal productivity. Let's use this insight to examine the incomes of two groups of workers: farmers and barbers.

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- a. Over the past century, the productivity of farmers has risen substantially because of technological progress. According to the neoclassical theory, what should have happened to their real wage?
 - b. In what units is the real wage discussed in part (a) measured?
 - c. Over the same period, the productivity of barbers has remained constant. What should have happened to their real wage?
 - d. In what units is the real wage in part (c) measured?
 - e. Suppose workers can move freely between being farmers and being barbers. What does this mobility imply for the wages of farmers and barbers?
 - f. What do your previous answers imply for the price of haircuts relative to the price of food?
 - g. Who benefits from technological progress in farming—farmers or barbers?
4. The government raises taxes by \$100 billion. If the marginal propensity to consume is 0.6, what happens to the following? Do they rise or fall? By what amounts?
- a. Public saving.
 - b. Private saving.
 - c. National saving.
 - d. Investment.
5. Suppose that an increase in consumer confidence raises consumers' expectations of future income and thus the amount they want to consume today. This might be interpreted as an upward shift in the consumption function. How does this shift affect investment and the interest rate?
6. Consider an economy described by the following equations:

$$Y = C + I + G,$$

$$Y = 5,000,$$

$$G = 1,000,$$

$$T = 1,000,$$

$$C = 250 + 0.75(Y - T),$$

$$I = 1,000 - 50r.$$

- a. In this economy, compute private saving, public saving, and national saving.
 - b. Find the equilibrium interest rate.
 - c. Now suppose that G rises to 1,250. Compute private saving, public saving, and national saving.
 - d. Find the new equilibrium interest rate.
7. Suppose that the government increases taxes and government purchases by equal amounts. What happens to the interest rate and investment in response to this balanced-budget change? Does your answer depend on the marginal propensity to consume?
8. When the government subsidizes investment, such as with an investment tax credit, the subsidy often applies to only some types of investment. This question asks you to consider the effect of such a change. Suppose there are two types of investment in the economy: business investment and residential investment. And suppose that the government institutes an investment tax credit only for business investment.
- a. How does this policy affect the demand curve for business investment? The demand curve for residential investment?
 - b. Draw the economy's supply and demand for loanable funds. How does this policy affect the supply and demand for loanable funds? What happens to the equilibrium interest rate?
 - c. Compare the old and the new equilibrium. How does this policy affect the total quantity of investment? The quantity of business investment? The quantity of residential investment?
9. If consumption depended on the interest rate, how would that affect the conclusions reached in this chapter about the effects of fiscal policy?



APPENDIX

The Cobb–Douglas Production Function

What production function describes how actual economies turn capital and labor into GDP? The answer to this question came from a historic collaboration between a U.S. senator and a mathematician.

Paul Douglas was a U.S. senator from Illinois from 1949 to 1966. In 1927, however, when he was still a professor of economics, he noticed a surprising fact: the division of national income between capital and labor had been roughly constant over a long period. In other words, as the economy grew more prosperous over time, the total income of workers and the total income of capital owners grew at almost exactly the same rate. This observation caused Douglas to wonder what conditions lead to constant factor shares.

Douglas asked Charles Cobb, a mathematician, what production function, if any, would produce constant factor shares if factors always earned their marginal products. The production function would need to have the property that

$$\text{Capital Income} = MPK \times K = \alpha Y$$

and

$$\text{Labor Income} = MPL \times L = (1 - \alpha) Y,$$

where α is a constant between zero and one that measures capital's share of income. That is, α determines what share of income goes to capital and what share goes to labor. Cobb showed that the function with this property is

$$Y = F(K, L) = AK^\alpha L^{1-\alpha},$$

where A is a parameter greater than zero that measures the productivity of the available technology. This function became known as the *Cobb–Douglas production function*.

Let's take a closer look at some of the properties of this production function. First, the Cobb–Douglas production function has constant returns to scale. That is, if capital and labor are increased by the same proportion, then output increases by that proportion as well.⁵

⁵ *Mathematical note:* To prove that the Cobb–Douglas production function has constant returns to scale, examine what happens when we multiply capital and labor by a constant z :

$$F(zK, zL) = A(zK)^\alpha (zL)^{1-\alpha}.$$

Expanding terms on the right,

$$F(zK, zL) = Az^\alpha K^\alpha z^{1-\alpha} L^{1-\alpha}.$$

(footnote continues)

Next, consider the marginal products for the Cobb–Douglas production function. The marginal product of labor is⁶

$$MPL = (1 - \alpha) AK^\alpha L^{-\alpha},$$

and the marginal product of capital is

$$MPK = \alpha AK^{\alpha-1} L^{1-\alpha}.$$

From these equations, recalling that α is between zero and one, we can see what causes the marginal products of the two factors to change. An increase in the amount of capital raises the MPL and reduces the MPK . Similarly, an increase in the amount of labor reduces the MPL and raises the MPK . A technological advance that increases the parameter A raises the marginal product of both factors proportionately.

The marginal products for the Cobb–Douglas production function can also be written as⁷

$$MPL = (1 - \alpha) Y/L.$$

$$MPK = \alpha Y/K.$$

The MPL is proportional to output per worker, and the MPK is proportional to output per unit of capital. Y/L is called *average labor productivity*, and Y/K is called *average capital productivity*. If the production function is Cobb–Douglas, then the marginal productivity of a factor is proportional to its average productivity.

We can now verify that if factors earn their marginal products, then the parameter α indeed tells us how much income goes to labor and how much goes to capital. The total wage bill, which we have seen is $MPL \times L$, is simply $(1 - \alpha)Y$.

Rearranging to bring like terms together, we get

$$F(zK, zL) = z^\alpha z^{1-\alpha} AK^\alpha L^{1-\alpha}.$$

Since $z^\alpha z^{1-\alpha} = z$, our function becomes

$$F(zK, zL) = zAK^\alpha L^{1-\alpha}.$$

But $AK^\alpha L^{1-\alpha} = F(K, L)$. Thus,

$$F(zK, zL) = zF(K, L) = zY.$$

Hence, the amount of output Y increases by the same factor z , which implies that this production function has constant returns to scale.

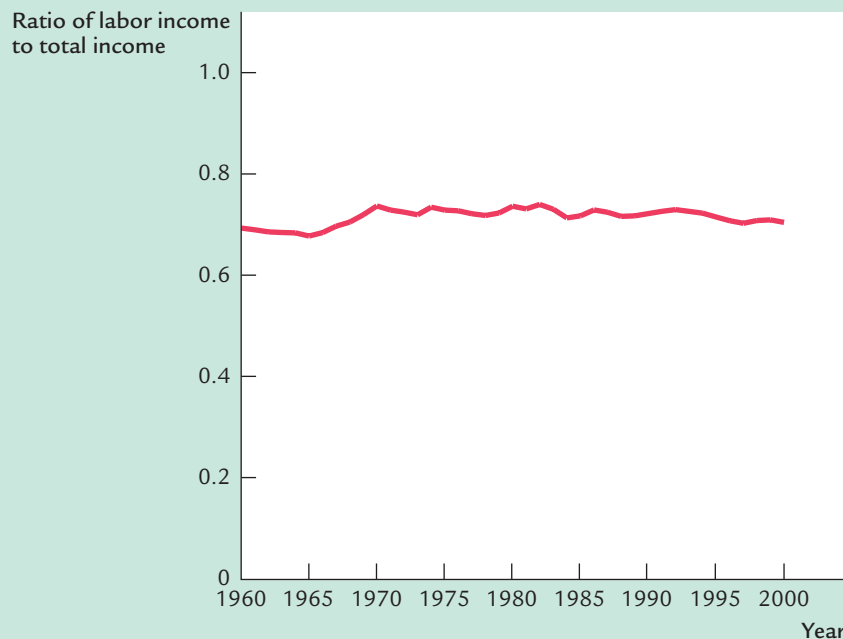
⁶ *Mathematical note:* Obtaining the formulas for the marginal products from the production function requires a bit of calculus. To find the MPL , differentiate the production function with respect to L . This is done by multiplying by the exponent $(1 - \alpha)$, and then subtracting 1 from the old exponent to obtain the new exponent, $-\alpha$. Similarly, to obtain the MPK , differentiate the production function with respect to K .

⁷ *Mathematical note:* To check these expressions for the marginal products, substitute in the production function for Y to show that these expressions are equivalent to the earlier formulas for the marginal products.

Therefore, $(1 - \alpha)$ is labor's share of output. Similarly, the total return to capital, $MPK \times K$, is αY , and α is capital's share of output. The ratio of labor income to capital income is a constant, $(1 - \alpha)/\alpha$, just as Douglas observed. The factor shares depend only on the parameter α , not on the amounts of capital or labor or on the state of technology as measured by the parameter A .

More recent U.S. data are also consistent with the Cobb–Douglas production function. Figure 3–13 shows the ratio of labor income to total income in the United States from 1960 to 2000. Despite the many changes in the economy over the past four decades, this ratio has remained about 0.7. This division of income is easily explained by a Cobb–Douglas production function in which the parameter α is about 0.3.

figure 3-13



The Ratio of Labor Income to Total Income Labor income has remained about 0.7 of total income over a long period of time. This approximate constancy of factor shares is evidence for the Cobb–Douglas production function. (This figure is produced from U.S. national income accounts data. Labor income is compensation of employees. Total income is the sum of labor income, corporate profits, net interest, rental income, and depreciation. Proprietors' income is excluded from these calculations, because it is a combination of labor income and capital income.)

Source: U.S. Department of Commerce.

MORE PROBLEMS AND APPLICATIONS

1. Suppose that the production function is Cobb–Douglas with parameter $\alpha = 0.3$.
 - a. What fractions of income do capital and labor receive?
 - b. Suppose that immigration raises the labor force by 10 percent. What happens to total output (in percent)? The rental price of capital? The real wage?
 - c. Suppose that a gift of capital from abroad raises the capital stock by 10 percent. What happens to total output (in percent)? The rental price of capital? The real wage?
 - d. Suppose that a technological advance raises the value of the parameter A by 10 percent. What happens to total output (in percent)? The rental price of capital? The real wage?
2. (This problem requires the use of calculus.) Consider a Cobb–Douglas production function with three inputs. K is capital (the number of machines), L is labor (the number of workers), and H is human capital (the number of college degrees among the workers). The production function is

$$Y = K^{1/3}L^{1/3}H^{1/3}.$$
 - a. Derive an expression for the marginal product of labor. How does an increase in the amount of human capital affect the marginal product of labor?
 - b. Derive an expression for the marginal product of human capital. How does an increase in the amount of human capital affect the marginal product of human capital?
 - c. What is the income share paid to labor? What is the income share paid to human capital? In the national income accounts of this economy, what share of total income do you think workers would appear to receive? (*Hint:* Consider where the return to human capital shows up.)
 - d. An unskilled worker earns the marginal product of labor, whereas a skilled worker earns the marginal product of labor plus the marginal product of human capital. Using your answers to (a) and (b), find the ratio of the skilled wage to the unskilled wage. How does an increase in the amount of human capital affect this ratio? Explain.
 - e. Some people advocate government funding of college scholarships as a way of creating a more egalitarian society. Others argue that scholarships help only those who are able to go to college. Do your answers to the preceding questions shed light on this debate?

